## Molecular Biophysics

## 2013/2014, lecturer: Martin Zápotocký

4 lectures:

1. Molecular interactions, states of matter, Gibbs rule of phases, properties of ideal gas
2. Transport phenomena, fluid mechanics
3. Classification of dispersion systems, properties of solutions
4. Other analytical dispersions; properties of colloids

Warning: figures and equations drawn on the blackboard are not included in these slides

## Lecture 1

## Molecular interactions

## Intra-molecular and inter-molecular interactions

- Distinguish interactions within molecules (e.g. covalent or ionic bonds) vs. interactions between molecules

- Intra-molecular interactions determine the structure of molecules; inter-molecular interactions determine the phase of matter (gas, liquid, solid, ...)
- Inter-molecular interactions are weaker than intra-molecular
- Bond energy: energy required to move the consituents very far from each other
- Intra-molecular bond energies (for 1 mole of molecules): typically $100-400 \mathrm{~kJ} / \mathrm{mol}$
- Typical inter-molecular interaction strength: up to several kJ/mol
- Hydrogen bonds: $10-45$ kJ/mol


## Types of inter-molecular forces

- Short-range repulsion + long-range attraction
- The long-range part is non-covalent, and is due to electrostatic interactions. Strongest between two ions (force $\mathrm{F} \sim r^{-2}$ ) or between ions and permanent dipole ( $\mathrm{F} \sim r^{-5}$, thermal averaging)
- Van der Waals forces are interactions involving neutral molecules, $\mathrm{F} \sim r^{-7}$

- Keesom force: between two permanent dipoles
- inductive force (Debye): involves an induced dipole
- dispersion force (London): between transitory dipoles (fluctuations).
 E.g. noble gases. Weakest force.


## Phases of matter

## Phases of matter

- A phase is a form of matter that is homogeneous in chemical composition and physical state
- The phase is characterized by state variables: pressure $p$, volume V , temperature T , composition, ...
- Examples of distinct phases: ice cubes in water; oil droplets in water
- All materials have the following 4 basic phases:
- solid
- liquid
- gas
- plasma
- In addition, some materials have mesomorphic phases (such as liquid crystals)
- The phase is determined by the competition of the weak intermolecular forces with thermal (disordering) effects


## Crystalline solids

- In a crystal, inter-atomic forces arrange the atoms into a spatial lattice: the atoms have fixed equilibrium positions
- Covalent, metallic, ionic, molecular crystals
- The details of inter-atomic interactions determine the type of lattice (cubic, hexagonal, ...)
- How do we know this? X-ray diffraction: can probe structures of scale comparable to the wavelength of X-ray photons (Angstroms).



## Crystalline solids

- The type of lattice (cubic, hexagonal, ...) determines the shape of grown crystals and their macroscopic properties
- Example: graphite (one of the phases of Carbon) - stack of planes with hexagonal order, can easily slide along the planes
- When temperature is increased, vibrations around equilibrium positions are more pronounced and the crystal lattice can melt
- Lindemann criterion: lattice will melt when amplitude of vibrations reaches $10 \%$ of intermolecular distance


Within graphite layers: covalent bonds

Between graphite layers:
Van der Waals interactions
Graphene: Nobel prize 2010


## Amorphous solids

- Materials such as glass or rubber have the mechanical properties of solids, but there is no crystalline lattice
- Atoms are rigidly bound, but only short-range lattice ordering may be found
- Glass can be prepared by quickly cooling below the
 crystallization temperature
- Amorphous solids as they have rigidity against shear. Some literature views them as extremely viscous fluids.


## Liquids

- Unlike solids, liquids flow. Molecules do not have fixed equilibrium positions.
- Liquids have surface tension: will tend to minimize the area of the interface with another liquid or with gas. Due to van der Waals forces.
- Most liquids are isotropic (properties do not depend on direction)
- Anisotropic liquids: have anisotropic molecules which tend to order in a common direction (orientational order vs. positional order).
- However, most liquids with anisotropic molecules (such as $\mathrm{H}_{2} \mathrm{O}$ ) do not have orientational order, and are isotropic.


## Types of liquid crystals

- Liquid crystals are anisotropic liquids
- Nematic LC: molecules have a common preferred direction
- Smectic LC: molecules have preferred direction and move within a preferred plane
- Cholesteric LC (chiral nematic): preferred direction rotates in a helix along the pitch
 axis
- Their anisotropic optical properties are used in LCDs (liquid crystal displays)


## Plasma

- At very high temperatures, gases ionize - the neutral molecules are split into ions and free electrons.
- Partial ionization above $1000^{\circ} \mathrm{C}$, full above $100000^{\circ} \mathrm{C}$
- Plasma can also be generated non-thermally: by ionizing radiation, or by high-intensity electric fields. In this case it is not in thermal equilibrium.


## Phase transitions

- A phase transition is a change in phase that occurs due to an external influence (heat, pressure, ...)

- Typical phase diagram:

- Example: boiling of water (liquid $\rightarrow$ gas) can occur due to increase in T or decrease in p


## Phases of water

- Hydrogen bond: an H atom bridges two electronegative atoms (typically O, N, F)
- Up to 4 bonds per molecule
- Results in high boiling point, melting point, and viscosity (compared to fluids without H-bonds)


- Triple point: ice, water, and vapor are in thermal equilibrium
- Critical point: water and vapor become indistinguishable (one phase)


## Gibbs rule of phases

- Applies to systems at thermodynamic equilibrium that contain multiple phases
- Follows from equality of chemical potentials (see lectures on thermodynamics)
- Gibbs rule:

$$
p+d=c+2
$$

$\mathrm{p}=$ number of phases
$\mathrm{c}=$ number of components
$\mathrm{d}=$ number of indendent state variables (number of degrees of freedom)

- Examples:
- water-ice coexistence: $\mathrm{p}=2$, $\mathrm{c}=1$, so $\mathrm{d}=1$
- water at triple point: $\mathrm{p}=3$, $\mathrm{c}=1$, so $\mathrm{d}=0$
- binary mixture: $\mathrm{c}=2, \mathrm{p} \geq 1$, so $\mathrm{d}=\mathrm{up}$ to 3 (relative concentration as additional degree of freedom)



## Properties of gases

## Gas phase

- A gas has no surface tension, and expands to fill the container
- There is no positional or orientational order of the molecules
- Ideal gas: is a theoretical construct, a gas of point-like molecules bouncing off each other elastically
- A gas is approximately ideal when:
- there are no van der Waals interactions
- average inter-molecular distance >> molecular size
- only translational degrees of freedom are active
- Satisfied well by the noble gases (Ar) in a wide range of p and T , and most other gases under typical conditions (but not close to condensation!)


## Equation of state for ideal gas

- Equation of state relates the basic state variables (in thermal equilibrium):

$$
p V=n R T
$$

- $\mathrm{R}=8.3 \mathrm{JK}^{-1} \mathrm{~mol}^{-1}$ is the universal gas constant
- n is the number of moles of the gas; 1 mole contains $N_{A}=$ $6.02 \times 10^{23}$ molecules (Avogadro constant)
- Alternative form of equation of state:

$$
p V=N k T
$$

- $N=n N_{A}$ is the total number of molecules
$-\mathrm{k}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$ is the Boltzmann constant
- Isothermal process: change in state that occurs at fixed T
- Boyle's law: $p V=$ const during an isothermal process
- Isobaric process: fixed p, $V / T=$ const
- Isochoric process: fixed V , $\mathrm{p} / T=$ const


## Maxwell-Boltzmann distribution

- Valid for ideal gas at thermal equilibrium
- Gives the proportion of molecules which have a given velocity v
- The most probable velocity: peak of the distribution, occurs for $v_{m p}=\sqrt{2 k T / m}$
- This is distinct from the mean (average) velocity,

$$
v_{a v}=\sqrt{8 k T / \pi m}
$$

- The root mean square velocity:

$$
v_{r m s}=\sqrt{3 k T / m}
$$

- Velocities increase with T, decrease with $m$


$$
f(v)=\sqrt{\frac{2}{\pi}\left(\frac{m}{k T}\right)^{3}} v^{2} \exp \left(\frac{-m v^{2}}{2 k T}\right)
$$

## Average energy per molecule

- Each molecule has kinetic
energy $E=\frac{1}{2} m v^{2}$
- Average kinetic energy of 1 molecule of ideal gas:

$$
\begin{gathered}
E_{a v}=\left\langle\frac{1}{2} m v^{2}\right\rangle=\frac{1}{2} m\left\langle v^{2}\right\rangle= \\
\frac{1}{2} m v_{r m s}^{2}=\frac{3}{2} k T
\end{gathered}
$$

- I.e., for each direction of motion (translational degree of freedom), the average energy is

$$
\frac{1}{2} k T
$$

- Translational vs rotational degrees of freedom:
- monoatomic molecule: $3+$ $0=3$
- rigid diatomic molecule: 3 $+2=5$
- rigid 3 (and more)-atomic: $3+3=6$
- non-rigid: in addition vibrational degrees of freedom


## Theorem of the equipartition of energy

- Maxwell's equipartition theorem: each degree of freedom has on average the energy

$$
\frac{1}{2} k T
$$

- Is valid not only for the ideal gas, but for any system in thermodynamic equilibrium
- Therefore, the total kinetic energy of 1 mole of diatomic gas is $\frac{5}{2} N_{A} k T=\frac{5}{2} R T$
- Note: memorize
$1 / 2 \mathrm{k}_{\mathrm{B}} \mathrm{T}=2.07 \times 10^{-21} \mathrm{~J}=0.013 \mathrm{eV}$ (at room temperature 300 K )
- This typical thermal energy scale should be always compared to other microscopic energy scales (e.g. electrostatic interactions) in the system


## Non-ideal gas

- Real gases have attractive intermolecular forces and a finite molecular size
- Improvement on the ideal gas equation: van der Waals equation of state (for 1 mole)

$$
\left(p+\frac{a}{V^{2}}\right)(V-b)=R T
$$

- Here b quantifies the volume of the molecules and a quantifies the van der Waals interactions
- Note: an ideal gas would never liquify! Van der Waals interactions are needed for transition into liquid phase.


## Lecture 2

## Microscopic vs. continuum description of a fluid (note: fluids = gases + liquids)

- Mean free path in a gas: average distance covered by a molecule between successive collisions
- $7 \mu \mathrm{~m}$ for air at standard conditions
- increases with temperature
- decreases with pressure

- In a liquid, mean free path is comparable to distance between molecules (and almost no free path for liquids with hydrogen bonds)
- Microscopic description of a fluid: in terms of positions and velocities of molecules (e.g. the Maxwell-Boltzmann distribution)
- Continuum description: in terms of local density, local average velocity (averaged over a small volume)
- can be used only if dimensions of interest >> mean free path


# Fluid statics 

## Pressure

- A fluid transmits a force uniformly in all directions
- Force acting on surface A in a fluid at rest:

$$
F=p A
$$

where p is the pressure; the force is normal to the surface

- Hydrostatic presure is due to the weight of fluid:

$$
p(h)=\rho g h+p_{a t m}
$$

- Units:
- in SI, 1 Pascal $=1$ Newton $/ \mathrm{m}^{2}$
$-1 \mathrm{~atm}=101325 \mathrm{~Pa}=760 \mathrm{~mm} \mathrm{Hg}=760$ torr
- $1 \mathrm{bar}=100000 \mathrm{~Pa}$
- Incompressible liquid: density does not change under applied force
- Pascal's principle for incompressible liquids: an increase in pressure is transmitted uniformly and isotropically to all parts of the liquid. Used in hydrostatic machines to amplify applied force.


## Surface tension

- Arises at the surface of a liquid. Molecules pulled inwards $\rightarrow$ higher cohesion at the surface.
- Force acting on a line segment $L$ in the surface:
- normal to the line, tangent to the surface
- magnitude $F=\sigma L, \sigma=$ surface tension

- for water at $20 \operatorname{deg} \mathrm{C}: \sigma=7.310^{-2} \mathrm{~N} / \mathrm{m}$. Decreases with T.
- In terms of energy: molecules at surface have fewer neighbors $\rightarrow$ cannot lower their energy as much as in bulk. Therefore, work is required to increase the surface of a liquid (to increase by $1 \mathrm{~m}^{2}$, need energy $=$ capillary constant $=\sigma$ )
- Small droplets are spherical, as a sphere minimizes surface area for given volume
- Bigger droplets are deformed due to gravitation and other forces. Note: mercury has 7 x larger $\sigma$ then water; glycerin slightly smaller $\sigma$.


## Surface tension examples



- Weight of needle balanced by water surface tension
- Insects walking on water
- Surfactants (soaps, detergents): materials composed of amphiphilic molecules. These concentrate at the liquid boundary, weaken interactions among water molecules $\rightarrow$ dramatic decrease in surface tension upon addition of detergent




## Laplace's law

- Relates the shape of a fluid boundary to the pressure difference $\Delta p$ across the boundary:

$$
\Delta p=\sigma\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)
$$

where $\sigma$ is the tension in the boundary,
$R_{1}$ and $R_{2}$ are the principal radii of curvature

- Reflects the balance of forces at the stationary boundary; p larger inside
- Example: a spherical water droplet of radius R has

$$
R_{1}=R_{2}=R, \sigma=\text { the surface tension, } \Delta p=2 \sigma / R
$$

- Example: a cylindrical blood vessel of radius R has

$$
R_{1}=R, R_{2}=\infty, \sigma=\text { elastic tension of vessel wall, } p=\sigma / R
$$

- E.g. for aorta, $\mathrm{p}=13 \mathrm{kPa} \rightarrow \sigma=170 \mathrm{~N} / \mathrm{m}$, for vein, $\mathrm{p}=2 \mathrm{kPa} \rightarrow \sigma=0.4 \mathrm{~N} / \mathrm{m}$ (see table in lecture notes)


## Fluid dynamics

## Equation of continuity

- Consider an incompressible fluid in a tube with cross-section A
- Flow rate Q : volume of fluid flowing through A per unit time,

$$
Q=\frac{\Delta V}{\Delta t}=\frac{A \Delta x}{\Delta t}=A v
$$

where $v$ is the velocity of the fluid

- Equation of continuity:
volume of fluid that enters $=$ volume of fluid that leaves

$$
A_{1} v_{1}=A_{2} v_{2}
$$



## Bernoulli's equation

- Equation of continuity gives the change in velocity. How to calculate the change in pressure?
- Conservation of energy: potential energy + kinetic energy $=$ const.
- Bernoulli's equation:

$$
p_{1}+\frac{1}{2} \rho v_{1}^{2}+\rho g h_{1}=p_{2}+\frac{1}{2} \rho v_{2}^{2}+\rho g h_{2}
$$



- Bernouilli‘s equation is valid only if there are no energy losses, i.e. friction can be neglected (ideal fluid)
- If fluid is not moving, we recover the equation for hydrostatic pressure


## Viscosity

- Real fluids have internal friction and dissipate energy (kinetic energy is transformed into heat)
- At contact of viscous fluid with a non-moving wall, velocity $=0$
- Parabolic velocity profile for slowly moving fluid in a tube:


$$
v=v_{\max }\left(1-\frac{r^{2}}{R^{2}}\right)
$$

- The velocity gradient leads to friction between the moving layers, resulting in tangential tension (shear stress) $\Sigma$ :

$$
\Sigma=\eta \frac{\Delta v}{\Delta r}
$$

- The constant $\eta$ is the dynamic viscosity, units Pa sec
- Kinematic viscosity: $\eta_{\text {kin }}=\eta / \rho$, units $\mathrm{m}^{2} / \mathrm{sec}$


## Hagen-Poiseuille law

- Ideal fluid: no driving force needed to maintain motion
- But for a viscous fluid, pressure decreases in the direction of flow. For a tube of length $L$ and radius $R$ :

$$
\Delta p=R_{f} Q
$$

where Q is the volume rate of flow, and the flow resistance $R_{f}$ is

$$
R_{f}=\frac{8 \eta L}{\pi R^{4}}
$$

- Note that for given $\Delta p$, the rate of flow increases very strongly with R !
- Ostwald viscometer:
- measurement of viscosity based on H-P law
- $\Delta p$ given by $\rho g L$, where L is the length of the capillary
- measure time needed for flow of known volume $\rightarrow \mathrm{Q} \rightarrow \eta$
- in practice, compare to standard liquid, ratio of times of flow $=$ ratio of kinematic viscosities


## Stokes law, Reynolds number

- Spherical object slowly moving relative to a fluid:
- Viscous forces result in drag force F on the object

$$
F=6 \pi \eta r v \quad(\text { Stokes law })
$$

where $r$ is the radius and $v$ the velocity of the object

- Falling object reaches terminal velocity when
gravitational force $=$ frictional force
- For low velocities, flow is laminar (sliding of layers)
- For high velocities, flow becomes turbulent (streamlines mix), HagenPoiseuille and Stokes laws no longer valid. Energy losses increase in turbulent flow. Heard as sound in stethoscope.
- Reynolds number: $R e=\frac{2 \rho v R}{\eta}$ (dimensionless)
- If $\mathrm{Re}<2000$ laminar flow, if $\mathrm{Re}>3000$ turbulent



## Typical viscosities and Reynolds numbers

- Air $\eta=0.02 \mathrm{mPa} \mathrm{s}$ (real gases are viscous!)
- Water 1 mPa s, blood 3 mPa s, glycerin 1 Pa s
- Viscosity increases with T in gases, decreases with T in liquids
- Typical boat: $\operatorname{Re}=10^{8}$, highly turbulent
- Blood flow in aorta: $\mathrm{Re}=1000-3000$, in vein 100 , in capillary 0.002
- Turbulence produces sound heard in stethoscope
- Moving bacteria: $\operatorname{Re}=10^{-5}$, no inertia


## Lecture 3

## Classification of dispersion systems

- Dispersion system: particles of one dispersive portion are dispersed in a continous dispersive medium
- Classification according to particle size:
- up to 1 nm , analytical dispersion (homogeneous)
- 1 nm to $1 \mu \mathrm{~m}$, colloidal dispersion
- above $1 \mu \mathrm{~m}$, coarse dispersion
- dispersion degree: $1 /$ size $\left(\mathrm{m}^{-1}\right)$
- Classification according to phase of dispersive portion and medium
- see table in lecture notes
- liquid in gas: vapours; aerosols; rain, fog
- gas in liquid: gas solution; foams; bubbles
- Common in living matter; pharmacy applications


## Properties of solutions

## Molecular diffusion

- Consider a particular molecule moving in a fluid
- It undergoes a random walk with average step size $=$ mean free path L
- Fundamental property of random walk:

rms displacement after N steps is proportional to $\sqrt{N}$ (not to N !)
- Therefore square of displacement of molecule after time $\tau$ is

$$
x_{r m s}^{2}=2 D \tau \quad \text { (in each direction) }
$$

- Diffusion coefficient D for spherical particle:

$$
D=\frac{k_{B} T}{6 \pi r \eta}
$$

- Typical values: e.g. haemoglobin in water $710^{-11} \mathrm{~m}^{2} \mathrm{~s}^{-1}$
- Time required to diffuse (in water) over distance $10 \mu \mathrm{~m}: 10 \mathrm{msec}$, over distance 1 cm : 1 hour!


## Transport by diffusion

- Diffusion tends to remove gradients in concentration
- Diffusion flux J: net number of moles crossing (in one direction) a unit area per unit time
- Fick's law of diffusion: diffusion flux is proportional to concentration gradient

$$
J=\frac{n}{A \tau}=-D \frac{\Delta c}{\Delta x}
$$

(net transport of molecules in direction opposite to concentration gradient)

- Transport by diffusion vs. transport by advection: diffusion effective only for short distances


## Transport of oxygen in breathing

- Human body (in rest) needs about 15 liters of $\mathrm{O}_{2}$ per hour
- Diffusion through skin is insufficient (only about $2 \%$ in humans). Sufficient only for small animals (below 0.5 cm , insects).
- Transport of air by convection into lungs, terminating in alveoli (cavities of diameter 0.1 mm to 0.3 mm ). Diffusion of $\mathrm{O}_{2}$ into blood across the walls of alveoli. Total surface area of alveoli $100 \mathrm{~m}^{2}$.
- How much air pressure is required to inflate the alveoli? Wall is coated with fluid; Laplace's law $\Delta p=2 \sigma / R$
- For $\sigma=7.310^{-2} \mathrm{~N} / \mathrm{m}$ (water), $\Delta p$ would be 3 atm ! Fluid on alveoli contains surfactants (lipids + proteins), reduces $\sigma$ to $10^{-3} \mathrm{~N} / \mathrm{m}$.
- Infant respiratory distress symptome:
 treatment with artificial surfactants


## Colligative properties of solutions

- Colligative properties depend only on the number (concentration) of solute particles. Do not depend on size, shape, or chemical properties.
- Proportional to molar concentration $C_{m}=\frac{N / N_{A}}{V}=\frac{c}{N_{A}}=\frac{c_{g}}{M}$
( $\mathrm{N}=$ number of particles, $c_{g}=$ mass concentration, $\mathrm{M}=$ molar mass)


## Osmotic pressure <br> Raoult's laws

## Osmotic pressure



- Osmosis: diffusion of water across a semi-permeable membrane (molecules of solute cannot pass through)
- In solution, water concentration is lowered due to solute $\rightarrow$ concentration gradient $\rightarrow$ diffusion of water molecules into the solution $\rightarrow$ increase in pressure in solution until flow is stopped
- The extra pressure in solution is the osmotic pressure
- Van't Hoff laws: $\quad P_{\text {osm }}=R T C_{m}$
- osmotic pressure proportional to molar concentration
- analytical solutions have higher osmotic pressure than colloids


## Hypertonic, hypotonic solutions

- Measuring osmotic pressure: can use hydrostatic pressure
- Two solutions with equal osmotic pressure are isotonic
- Hypertonic solution has greater amount of impermeable solute than cytosol. Draws water out of cells. In hypotonic solution, cells swell.
- Osmotic pressure crucial in cell physiology. Plants keep erect thanks to osmotic pressure (turgor pressure acting on cell wall).
- Red blood cells in pure water rupture (haemolysis) before the equilibrium osmotic pressure 800 kPa is reached



## Osmosis and water transport in the body



- Arterial capillary: blood pressure > osmotic pressure, net water transport out of the capillary
- Venous capillary: blood pressure < osmotic pressure, net water intake
- Kidney: removal of waste products, regulation of water volume
- Regulation of osmotic pressure by excretion of urine (hypertonic compared to blood)


## Hemodialysis

- Dialysis: replaces kidney function (except for hormone production)
- Blood and dialysis solution separated by membrane permeable to metabolites and ions, impermeable to proteins and blood cells.
- Diffusion of metabolites into dialysis solution
- Ultrafiltration (water removal / intake due to pressure difference)



## Raoult's laws

- Boiling temperature (at given pressure) is higher for solution than for pure solvent and is proportional to the molar concentration $C_{m}$ of solute

$$
\Delta T_{b}=K_{e} C_{m} \quad\left(K_{e}=\text { ebullioscopic constant }\right)
$$

- Freezing temperature (at given pressure) is lower for solution:

$$
\Delta T_{b}=-K_{c} C_{m} \quad\left(K_{c}=\text { cryoscopic constant }\right)
$$

- Pressure of vapors for a solution is lower than for pure solvent and is proportional to the number of particles dissolved: $\quad \frac{\Delta p}{p_{o}}=\frac{n_{2}}{n_{1}+n_{2}}$



## Adsorption at a boundary

- Adsorption is the adhesion of solute at the fluid boundary (moves within a thin layer)
- An increase in concentration $(\Delta c)$ of surfactant decreases the surface tension $(\Delta \sigma)$ of solution

- Tendency to decrease interfacial energy is balanced with diffusion, results in the Gibb's adsorption equilibrium equation:

$$
\Gamma=-\frac{c}{R T} \frac{\Delta \sigma}{\Delta c}
$$

where $\Gamma$ is the surface concentration (in $\mathrm{mol} / \mathrm{m}^{2}$ )

- A sufactant solute has $\frac{\Delta \sigma}{\Delta c}<0, \Gamma>0$. The value $\frac{\Delta \sigma}{\Delta c}$ is different for different surfactants, i.e. $\Gamma$ is not
 a colligative property


## Lecture 4

## Other analytical dispersions

## Mixture of gases

- Partial pressure of a gas in a gas mixture (with given $\mathrm{p}, \mathrm{V}, \mathrm{T}$ ) is the pressure the gas would have alone, if maintained at the same $\mathrm{V}, \mathrm{T}$
- Partial volume of a gas in a gas mixture is the volume it would have alone at the same $\mathrm{p}, \mathrm{T}$
- Dalton's law: the pressure of a gas mixture is the sum of partial pressures of its components (fully valid only for ideal gases)
- Amagad's law: the volume of a gas mixture is the sum of partial volumes of its components


## Vapor

- Vapor pressure is the pressure at which vapor is in thermodynamic equilibrium with the liquid phase
- Vapor pressure increases with temperature. The liquid boils when its vapor pressure reaches the surrounding pressure
- Saturated vapor: any extra amount of vapor will condense
- Absolute humidity: mass of water vapor in unit volume of air
- Maximum humidity: mass of saturated vapor in unit volume. E.g. at 1 atm and $20^{\circ} \mathrm{C}, \varphi=17 \mathrm{~g} / \mathrm{m}^{3}$; at $30^{\circ} \mathrm{C}, \varphi=30 \mathrm{~g} / \mathrm{m}^{3}$
- Relative air humidity: absolute humidity / maximum humidity
- Air at at $20^{\circ} \mathrm{C}$, saturated with water vapor: has partial pressures 23 mbar of water vapor, 780 mbar of nitrogen, 210 mbar of oxygen and 9 mbar of argon.


## Gas in liquid

- Gas in contact with a liquid (any liquid with which it does not react chemically): dissolves in the liquid, dynamic equilibrium of gas $\rightarrow$ liquid and liquid $\rightarrow$ gas
- Henry‘s law: amount of gas dissolved in liquid is proportional to the partial pressure of the gas above liquid
- Molar concentration of gas in liquid:

$$
c=\alpha_{B} p, \quad \alpha_{B}=\text { Bunsen's absorption coefficient }
$$

- Solubility decreases with increasing T
- Water at $20^{\circ} \mathrm{C}$ : for oxygen $\alpha_{B}=0.031$, for nitrogen $\alpha_{B}=0.016$. I.e. in water, higher amount of oxygen relative to nitrogen (compared to air)
- Nitrogen embolism: when air inhaled at high $\mathrm{p} \rightarrow$ high amount of nitrogen dissolved $\rightarrow$ released when returned to atmosperic $p \rightarrow$ nitrogen bubbles in capillaries, fatal if in brain. Need slow return to surface or decompression chamber.


## Solid in liquid

- Compounds that dissolve easily in polar solvent (water is strongly polar, dielectric $\epsilon_{r}=80$ ):
- dissociable into ions (e.g. NaCl )
- organic compounds with polar groups
- Solubility of solids increases with increasing T
- If solution is saturated, any additional solid will form precipitate
- Can precipitate by lowering temperature

- Oversaturated solution can be obtained when solidification centers (e.g. impurities) are absent from solution


## Properties of colloids

## Colloidal dispersions

- Dispersive portion consists of particles of size 1 nm to $1 \mu \mathrm{~m}$
- Solid in liquid: sol; liquid in liquid: colloidal emulsion
- Examples: ink; milk
- Lyophilic colloids: disperse spontaneously, colloid has lower energy (solvation decreases surface tension). E.g. starch, protein in water.
- Lyophobic colloids are unstable (hydrophobic molecules aggregate). E.g. emulsion of oil in water; metal compounds in water.
- Tyndall effect: intense light scattering from a colloid, with intensity proportional to concentration. Most intense scattering for shorter wavelengths $\rightarrow$ blue color.


## Electric double layer of colloidal particles

- Plays crucial role in stabilization of colloids
- Colloidal particles (macromolecules, micelles) have attractive Van der Waals interactions. If no repulsion, particles may coagulate.
- Repulsive Coulomb interaction due to electric double layer:
- negatively charged ions from electrolyte dispersive medium are adsorped on the surface of particles
- these attract positively charged counterions
- overall, the double layer is neutral
- slipping plane: part of the double layer that moves with the particle

- zeta potential $\zeta$ : value of electrostatic potential at the slipping plane. Good stability of colloid for $\zeta \geq 40 \mathrm{mV}$.



## Electrophoresis

- In a uniform electric field $E$, the colloidal particles will move relative to the fluid dispersive medium, with velocity

$$
v=\mu_{e} E
$$

where $\mu_{\mathrm{e}}$ is the electrophoretic mobility

- Smoluchowski equation: $\mu_{e}=\epsilon_{r} \epsilon_{o} \zeta / \eta$, where $\epsilon_{r}$ is the dielectric constant of fluid, $\eta$ is the viscosity of fluid
- Zeta potential $\zeta$ also called electrokinetic potential
- Distinct colloidal particles have different $\mu_{\mathrm{e}}$, allows separation of particles (e.g. separation of albumin and globulin in blood serum)


## Brownian motion

- Colloidal particles in a liquid or gas exhibit Brownian motion
- Due to imbalances of impact forces of molecules on the particle
- Understood before direct observation of molecules (Einstein, Smoluchowski)
- Kinetic energy of 1 collidal particle is of the order of $k_{B} T$



## Sedimentation

- Due to gravity, particles of size r and density $\rho_{0}$ move through fluid with terminal velocity v
- For spherical particles: Stokes force $6 \pi \eta r v$ balanced with buoyancy force $(4 / 3) \pi r^{3}\left(\rho-\rho_{0}\right) g$, which gives the sedimentation velocity

$$
v=\frac{2\left(\rho-\rho_{0}\right) g r^{2}}{9 \eta}
$$

- In equilibrium, a profile of colloid concentration is established, balance of sedimentation and diffusion
- Time needed to sediment: fast for coarse solutions, effectively infinite for analytical solutions
- Sedimentation rate test: aggregates of erythrocytes are formed during inflammation and sediment faster (bigger $r$ ).
- Centrifugation: enhanced sedimentation due to increased $g$ (inertial force). Used e.g. to efficiently separate blood components.

