Molecular Biophysics

2013/2014, lecturer: Martin Zápotocký

4 lectures:

- Molecular interactions, states of matter, Gibbs rule of phases, properties of ideal gas
- 2. Transport phenomena, fluid mechanics
- 3. Classification of dispersion systems, properties of solutions
- 4. Other analytical dispersions; properties of colloids

Warning: figures and equations drawn on the blackboard are not included in these slides

Lecture 1

Molecular interactions

Intra-molecular and inter-molecular interactions

• Distinguish interactions within molecules (e.g. covalent or ionic bonds) vs. interactions between molecules



• Intra-molecular interactions determine the structure of molecules; inter-molecular interactions determine the phase of matter (gas, liquid, solid, ...)

- Inter-molecular interactions are weaker than intra-molecular
- Bond energy: energy required to move the consituents very far from each other
- Intra-molecular bond energies (for 1 mole of molecules): typically 100 – 400 kJ/mol
- Typical inter-molecular interaction strength: up to several kJ/mol
- Hydrogen bonds: 10 45 kJ/mol

Types of inter-molecular forces

- Short-range repulsion + long-range attraction
- The long-range part is non-covalent, and is due to electrostatic interactions. Strongest between two ions (force $F \sim r^{-2}$) or between ions and permanent dipole ($F \sim r^{-5}$, thermal averaging)
- Van der Waals forces are interactions involving neutral molecules, $F \sim r^{-7}$
 - Keesom force:
 between two permanent dipoles
 - inductive force (Debye):
 involves an induced dipole
 - dispersion force (London):
 between transitory dipoles (fluctuations).
 E.g. noble gases. Weakest force.





Phases of matter

Phases of matter

- A phase is a form of matter that is homogeneous in chemical composition and physical state
- The phase is characterized by state variables: pressure p, volume V, temperature T, composition, ...
- Examples of distinct phases: ice cubes in water; oil droplets in water

- All materials have the following 4 basic phases:
 - solid
 - liquid
 - gas
 - plasma
- In addition, some materials have mesomorphic phases (such as liquid crystals)
- The phase is determined by the competition of the weak intermolecular forces with thermal (disordering) effects

Crystalline solids

- In a crystal, inter-atomic forces arrange the atoms into a spatial lattice: the atoms have fixed equilibrium positions
- Covalent, metallic, ionic, molecular crystals
- The details of inter-atomic interactions determine the type of lattice (cubic, hexagonal, ...)
- How do we know this? X-ray diffraction: can probe structures of scale comparable to the wavelength of X-ray photons (Angstroms).





Crystalline solids

- The type of lattice (cubic, hexagonal, ...) determines the shape of grown crystals and their macroscopic properties
- Example: graphite (one of the phases of Carbon) stack of planes with hexagonal order, can easily slide along the planes
- When temperature is increased, vibrations around equilibrium positions are more pronounced and the crystal lattice can melt
- Lindemann criterion: lattice will melt when amplitude of vibrations reaches 10% of intermolecular distance



Within graphite layers: covalent bonds

Between graphite layers: Van der Waals interactions Graphene: Nobel prize 2010



Amorphous solids

- Materials such as glass or rubber have the mechanical properties of solids, but there is no crystalline lattice
- Atoms are rigidly bound, but only short-range lattice ordering may be found
- Glass can be prepared by quickly cooling below the crystallization temperature
- Amorphous solids as they have rigidity against shear. Some literature views them as extremely viscous fluids.



Liquids

- Unlike solids, liquids flow. Molecules do not have fixed equilibrium positions.
- Liquids have surface tension: will tend to minimize the area of the interface with another liquid or with gas. Due to van der Waals forces.
- Most liquids are isotropic (properties do not depend on direction)

- Anisotropic liquids: have anisotropic molecules which tend to order in a common direction (orientational order vs. positional order).
- However, most liquids with anisotropic molecules (such as H₂O) do not have orientational order, and are isotropic.

Types of liquid crystals

- Liquid crystals are anisotropic liquids
- Nematic LC: molecules have a common preferred direction
- Smectic LC: molecules have preferred direction and move within a preferred plane
- Cholesteric LC (chiral nematic): preferred direction rotates in a helix along the pitch axis
- Their anisotropic optical properties are used in LCDs (liquid crystal displays)



Plasma

- At very high temperatures, gases ionize – the neutral molecules are split into ions and free electrons.
- Partial ionization above 1000 °C, full above 100 000 °C
- Plasma can also be generated non-thermally: by ionizing radiation, or by high-intensity electric fields. In this case it is not in thermal equilibrium.

Phase transitions

• A phase transition is a change in phase that occurs due to an external influence (heat, pressure, ...)



• Typical phase diagram:



 Example: boiling of water (liquid → gas) can occur due to increase in T or decrease in p

Phases of water

- Hydrogen bond: an H atom bridges two electronegative atoms (typically O, N, F)
- Up to 4 bonds per molecule
- Results in high boiling point, melting point, and viscosity (compared to fluids without H-bonds)





- Triple point: ice, water, and vapor are in thermal equilibrium
- Critical point: water and vapor become indistinguishable (one phase)

Gibbs rule of phases

- Applies to systems at thermodynamic equilibrium that contain multiple phases
- Follows from equality of chemical potentials (see lectures on thermodynamics)
- Gibbs rule:

p+d=c+2

p = number of phasesc = number of components

d = number of indendent state variables (number of degrees of freedom)

- Examples:
 - water-ice coexistence: p=2,
 c=1, so d=1
 - water at triple point: p=3, c=1, so d=0
 - binary mixture: $c = 2, p \ge 1$, so d = up to 3 (relative concentration as additional degree of freedom)



Properties of gases

Gas phase

- A gas has no surface tension, and expands to fill the container
- There is no positional or orientational order of the molecules
- Ideal gas: is a theoretical construct, a gas of point-like molecules bouncing off each other elastically

- A gas is approximately ideal when:
 - there are no van der Waals interactions
 - average inter-molecular distance >> molecular size
 - only translational degrees of freedom are active
- Satisfied well by the noble gases (Ar) in a wide range of p and T, and most other gases under typical conditions (but not close to condensation!)

Equation of state for ideal gas

• Equation of state relates the basic state variables (in thermal equilibrium):

pV = nRT

- $R = 8.3 JK^{-1} mol^{-1}$ is the universal gas constant
- n is the number of moles of the gas; 1 mole contains $N_A =$ 6.02×10^{23} molecules (Avogadro constant)

• Alternative form of equation of state:

$$pV = NkT$$

- $N = nN_A$ is the total number of molecules
- $k = 1.38 \times 10^{-23} \text{ J/K}$ is the Boltzmann constant
- Isothermal process: change in state that occurs at fixed T
- Boyle's law: *pV* = const during an isothermal process
- Isobaric process: fixed p, V/T = const
- Isochoric process: fixed V,
 p/T = const

Maxwell-Boltzmann distribution

- Valid for ideal gas at thermal equilibrium
- Gives the proportion of molecules which have a given velocity v
- The most probable velocity: peak of the distribution, occurs for $v_{mp} = \sqrt{2kT/m}$
- This is distinct from the mean (average) velocity, $m = \sqrt{\Omega kT/\pi m}$

$$v_{av} = \sqrt{8kT/\pi m}$$

• The root mean square velocity: $v_{rms} = \sqrt{3kT/m}$ • Velocities increase with T, decrease with m



$$f(v) = \sqrt{\frac{2}{\pi} \left(\frac{m}{kT}\right)^3} v^2 \exp\left(\frac{-mv^2}{2kT}\right)$$

Average energy per molecule

- Each molecule has kinetic energy $E = \frac{1}{2}mv^2$
- Average kinetic energy of 1 molecule of ideal gas:

$$E_{av} = \left\langle \frac{1}{2} m v^2 \right\rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} kT$$

• I.e., for each direction of motion (translational degree of freedom), the average energy is

$$\frac{1}{2}kT$$

- Translational vs rotational degrees of freedom:
 - monoatomic molecule: 3 + 0 = 3
 - rigid diatomic molecule: 3 + 2 = 5
 - rigid 3 (and more)-atomic: 3+3=6
 - non-rigid: in addition
 vibrational degrees of
 freedom

Theorem of the equipartition of energy

• Maxwell's equipartition theorem: each degree of freedom has on average the energy

$$\frac{1}{2}kT$$

- Is valid not only for the ideal gas, but for any system in thermodynamic equilibrium
- Therefore, the total kinetic energy of 1 mole of diatomic gas is $\frac{5}{2}N_AkT = \frac{5}{2}RT$

• Note: memorize

 $\frac{1}{2} k_{\rm B} T = 2.07 \times 10^{-21} \text{ J} = 0.013 \text{ eV}$ (at room temperature 300 K)

• This typical thermal energy scale should be always compared to other microscopic energy scales (e.g. electrostatic interactions) in the system

Non-ideal gas

- Real gases have attractive intermolecular forces and a finite molecular size
- Improvement on the ideal gas equation: van der Waals equation of state (for 1 mole)

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

• Here b quantifies the volume of the molecules and a quantifies the van der Waals interactions

• Note: an ideal gas would never liquify! Van der Waals interactions are needed for transition into liquid phase.

Lecture 2

Microscopic vs. continuum description of a fluid (note: fluids = gases + liquids)

- Mean free path in a gas: average distance covered by a molecule between successive collisions
 - $-7 \mu m$ for air at standard conditions
 - increases with temperature
 - decreases with pressure
- In a liquid, mean free path is comparable to distance between molecules (and almost no free path for liquids with hydrogen bonds)
- Microscopic description of a fluid: in terms of positions and velocities of molecules (e.g. the Maxwell-Boltzmann distribution)
- Continuum description: in terms of local density, local average velocity (averaged over a small volume)
 - can be used only if dimensions of interest >> mean free path



Fluid statics

Pressure

- A fluid transmits a force uniformly in all directions
- Force acting on surface A in a fluid at rest:

F = pA

where p is the pressure; the force is normal to the surface

• Hydrostatic presure is due to the weight of fluid:

 $p(h) = \rho g h + p_{atm}$

- Units:
 - in SI, 1 Pascal = 1 Newton / m^2
 - -1 atm = 101 325 Pa = 760 mm Hg = 760 torr
 - -1 bar = 100 000 Pa
- Incompressible liquid: density does not change under applied force
- Pascal's principle for incompressible liquids: an increase in pressure is transmitted uniformly and isotropically to all parts of the liquid. Used in hydrostatic machines to amplify applied force.

Surface tension

- Arises at the surface of a liquid. Molecules pulled inwards
 → higher cohesion at the surface.
- Force acting on a line segment L in the surface:
 - normal to the line, tangent to the surface
 - magnitude $F = \sigma L$, $\sigma =$ surface tension
 - for water at 20 deg C: $\sigma = 7.3 \ 10^{-2} \text{ N/m}$. Decreases with T.
- In terms of energy: molecules at surface have fewer neighbors \rightarrow cannot lower their energy as much as in bulk. Therefore, work is required to increase the surface of a liquid (to increase by 1 m², need energy = capillary constant = σ)
- Small droplets are spherical, as a sphere minimizes surface area for given volume
- Bigger droplets are deformed due to gravitation and other forces. Note: mercury has 7 x larger σ then water; glycerin slightly smaller σ .



Surface tension examples





- Weight of needle balanced by water surface tension
- Insects walking on water
- Surfactants (soaps, detergents): materials composed of amphiphilic molecules. These concentrate at the liquid boundary, weaken interactions among water molecules → dramatic decrease in surface tension upon addition of detergent





Laplace's law

• Relates the shape of a fluid boundary to the pressure difference Δp across the boundary:

$$\Delta p = \sigma(\frac{1}{R_1} + \frac{1}{R_2})$$

where σ is the tension in the boundary,

 R_1 and R_2 are the principal radii of curvature

- Reflects the balance of forces at the stationary boundary; p larger inside
- Example: a spherical water droplet of radius R has

 $R_1 = R_2 = R$, σ = the surface tension, $\Delta p = 2 \sigma/R$

• Example: a cylindrical blood vessel of radius R has

 $R_1 = R, R_2 = \infty, \sigma =$ elastic tension of vessel wall, $p = \sigma/R$

• E.g. for aorta, p=13 kPa $\rightarrow \sigma$ =170 N/m, for vein, p=2 kPa $\rightarrow \sigma$ =0.4 N/m (see table in lecture notes)

Fluid dynamics

Equation of continuity

- Consider an incompressible fluid in a tube with cross-section A
- Flow rate Q: volume of fluid flowing through A per unit time,

$$Q = \frac{\Delta V}{\Delta t} = \frac{A\Delta x}{\Delta t} = Av$$

where v is the velocity of the fluid

• Equation of continuity:

volume of fluid that enters = volume of fluid that leaves

$$A_1v_1 = A_2v_2$$



Bernoulli's equation

- Equation of continuity gives the change in velocity. How to calculate the change in pressure?
- Conservation of energy: potential energy + kinetic energy = const.
- Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$



- Bernouilli's equation is valid only if there are no energy losses, i.e. friction can be neglected (ideal fluid)
- If fluid is not moving, we recover the equation for hydrostatic pressure

Viscosity

- Real fluids have internal friction and dissipate energy (kinetic energy is transformed into heat)
- At contact of viscous fluid with a non-moving wall, velocity = 0
- Parabolic velocity profile for slowly moving fluid in a tube:



$$v = v_{max} (1 - \frac{r^2}{R^2})$$

• The velocity gradient leads to friction between the moving layers, resulting in tangential tension (shear stress) Σ :

$$\Sigma = \eta \, \frac{\Delta \nu}{\Delta r}$$

- The constant η is the dynamic viscosity, units Pa sec
- Kinematic viscosity: $\eta_{kin} = \eta/\rho$, units m²/sec

Hagen-Poiseuille law

- Ideal fluid: no driving force needed to maintain motion
- But for a **viscous** fluid, pressure decreases in the direction of flow. For a tube of length L and radius R:

$$\Delta p = R_f Q$$

where Q is the volume rate of flow, and the flow resistance R_f is

$$R_f = \frac{8\eta L}{\pi R^4}$$

- Note that for given Δp , the rate of flow increases very strongly with R!
- Ostwald viscometer:
 - measurement of viscosity based on H-P law
 - Δp given by ρgL , where L is the length of the capillary
 - measure time needed for flow of known volume $\rightarrow Q \rightarrow \eta$
 - in practice, compare to standard liquid, ratio of times of flow = ratio of kinematic viscosities



Stokes law, Reynolds number

- Spherical object slowly moving relative to a fluid:
- Viscous forces result in drag force F on the object

 $F = 6\pi\eta r\nu$ (Stokes law) where r is the radius and v the velocity of the object

• Falling object reaches terminal velocity when gravitational force = frictional force



- For low velocities, flow is laminar (sliding of layers)
- For high velocities, flow becomes turbulent (streamlines mix), Hagen-Poiseuille and Stokes laws no longer valid. Energy losses increase in turbulent flow. Heard as sound in stethoscope.
- Reynolds number: $Re = \frac{2\rho vR}{\eta}$ (dimensionless)
- If Re < 2000 laminar flow, if Re > 3000 turbulent



High velocity with turbulence

Typical viscosities and Reynolds numbers

- Air $\eta = 0.02$ mPa s (real gases are viscous!)
- Water 1 mPa s, blood 3 mPa s, glycerin 1 Pa s
- Viscosity increases with T in gases, decreases with T in liquids
- Typical boat: $Re = 10^8$, highly turbulent
- Blood flow in aorta: Re = 1000 3000, in vein 100, in capillary 0.002
- Turbulence produces sound heard in stethoscope
- Moving bacteria: $Re = 10^{-5}$, no inertia

Lecture 3

Classification of dispersion systems

- Dispersion system: particles of one dispersive portion are dispersed in a continous dispersive medium
- Classification according to particle size:
 - up to 1 nm, analytical dispersion (homogeneous)
 - -1 nm to 1 μ m, colloidal dispersion
 - above 1 μ m, coarse dispersion
 - dispersion degree: $1/\text{size}(m^{-1})$
- Classification according to phase of dispersive portion and medium
 - see table in lecture notes
 - liquid in gas: vapours; aerosols; rain, fog
 - gas in liquid: gas solution; foams; bubbles
- Common in living matter; pharmacy applications

Properties of solutions

Molecular diffusion

- Consider a particular molecule moving in a fluid
- It undergoes a random walk with average step size = mean free path L
- Fundamental property of random walk: rms displacement after N steps is proportional to \sqrt{N} (not to N!)
- Therefore square of displacement of molecule after time τ is

 $x_{rms}^2 = 2D\tau$ (in each direction)

• Diffusion coefficient D for spherical particle:

$$D = \frac{k_B T}{6\pi \ r \ \eta}$$

- Typical values: e.g. haemoglobin in water 7 $10^{-11} m^2 s^{-1}$
- Time required to diffuse (in water) over distance 10 µm: 10 msec, over distance 1 cm: 1 hour!



wall

Transport by diffusion

- Diffusion tends to remove gradients in concentration
- Diffusion flux J: net number of moles crossing (in one direction) a unit area per unit time
- Fick's law of diffusion: diffusion flux is proportional to concentration gradient

$$J = \frac{n}{A\tau} = -D \frac{\Delta c}{\Delta x}$$

(net transport of molecules in direction opposite to concentration gradient)

• Transport by diffusion vs. transport by advection: diffusion effective only for short distances

Transport of oxygen in breathing

- Human body (in rest) needs about 15 liters of O_2 per hour
- Diffusion through skin is insufficient (only about 2% in humans). Sufficient only for small animals (below 0.5 cm, insects).
- Transport of air by convection into lungs, terminating in alveoli (cavities of diameter 0.1 mm to 0.3 mm). Diffusion of O_2 into blood across the walls of alveoli. Total surface area of alveoli 100 m².
- How much air pressure is required to inflate the alveoli? Wall is coated with fluid; Laplace's law $\Delta p = 2 \sigma/R$
- For $\sigma = 7.3 \ 10^{-2} \ \text{N/m}$ (water), Δp would be 3 atm! Fluid on alveoli contains surfactants (lipids + proteins), reduces σ to $10^{-3} \ \text{N/m}$.
- Infant respiratory distress symptome: treatment with artificial surfactants



Colligative properties of solutions

- Colligative properties depend only on the number (concentration) of solute particles. Do not depend on size, shape, or chemical properties.
- Proportional to molar concentration $C_m = \frac{N/N_A}{V} = \frac{c}{N_A} = \frac{c_g}{M}$

(N = number of particles, c_g = mass concentration, M = molar mass)

Osmotic pressure Raoult's laws

Osmotic pressure



- Osmosis: diffusion of water across a semi-permeable membrane (molecules of solute cannot pass through)
- In solution, water concentration is lowered due to solute → concentration gradient → diffusion of water molecules into the solution → increase in pressure in solution until flow is stopped
- The extra pressure in solution is the osmotic pressure
- Van't Hoff laws: $P_{osm} = RTC_m$
 - osmotic pressure proportional to molar concentration
 - analytical solutions have higher osmotic pressure than colloids

Hypertonic, hypotonic solutions

- Measuring osmotic pressure: can use hydrostatic pressure
- Two solutions with equal osmotic pressure are isotonic
- Hypertonic solution has greater amount of impermeable solute than cytosol. Draws water out of cells. In hypotonic solution, cells swell.
- Osmotic pressure crucial in cell physiology. Plants keep erect thanks to osmotic pressure (turgor pressure acting on cell wall).
- Red blood cells in pure water rupture (haemolysis) before the equilibrium osmotic pressure 800 kPa is reached



Osmosis and water transport in the body



- Arterial capillary: blood pressure > osmotic pressure, net water transport out of the capillary
- Venous capillary: blood pressure < osmotic pressure, net water intake
- Kidney: removal of waste products, regulation of water volume
- Regulation of osmotic pressure by excretion of urine (hypertonic compared to blood)

Hemodialysis

- Dialysis: replaces kidney function (except for hormone production)
- Blood and dialysis solution separated by membrane permeable to metabolites and ions, impermeable to proteins and blood cells.
- Diffusion of metabolites into dialysis solution
- Ultrafiltration (water removal / intake due to pressure difference)



Raoult's laws

• Boiling temperature (at given pressure) is higher for solution than for pure solvent and is proportional to the molar concentration C_m of solute

 $\Delta T_b = K_e C_m$ (*K_e*= ebullioscopic constant)

• Freezing temperature (at given pressure) is lower for solution:

 $\Delta T_b = -K_c C_m \qquad (K_c = \text{cryoscopic constant})$

• Pressure of vapors for a solution is lower than for pure solvent and is proportional to the number of particles dissolved: $\frac{\Delta p}{p_0} = \frac{n_2}{n_1 + n_2}$



Adsorption at a boundary

- Adsorption is the adhesion of solute at the fluid boundary (moves within a thin layer)
- An increase in concentration (Δc) of surfactant decreases the surface tension ($\Delta \sigma$) of solution
- Tendency to decrease interfacial energy is balanced with diffusion, results in the Gibb's adsorption equilibrium equation:

$$\Gamma = -\frac{c}{RT}\frac{\Delta\sigma}{\Delta c}$$

where Γ is the surface concentration (in mol/m²)

• A sufactant solute has $\frac{\Delta\sigma}{\Delta c} < 0$, $\Gamma > 0$. The value $\frac{\Delta\sigma}{\Delta c}$ is different for different surfactants, i.e. Γ is not a colligative property





Lecture 4

Other analytical dispersions

Mixture of gases

- Partial pressure of a gas in a gas mixture (with given p, V, T) is the pressure the gas would have alone, if maintained at the same V, T
- Partial volume of a gas in a gas mixture is the volume it would have alone at the same p, T
- Dalton's law: the pressure of a gas mixture is the sum of partial pressures of its components (fully valid only for ideal gases)
- Amagad's law: the volume of a gas mixture is the sum of partial volumes of its components

Vapor

- Vapor pressure is the pressure at which vapor is in thermodynamic equilibrium with the liquid phase
- Vapor pressure increases with temperature. The liquid boils when its vapor pressure reaches the surrounding pressure
- Saturated vapor: any extra amount of vapor will condense
- Absolute humidity: mass of water vapor in unit volume of air
- Maximum humidity: mass of saturated vapor in unit volume. E.g. at 1atm and 20 °C, $\varphi = 17$ g/m³; at 30 °C, $\varphi = 30$ g/m³
- Relative air humidity: absolute humidity / maximum humidity
- Air at at 20 °C, saturated with water vapor: has partial pressures
 23 mbar of water vapor, 780 mbar of nitrogen, 210 mbar of oxygen and 9 mbar of argon.

Gas in liquid

- Gas in contact with a liquid (any liquid with which it does not react chemically): dissolves in the liquid, dynamic equilibrium of gas → liquid and liquid → gas
- Henry's law: amount of gas dissolved in liquid is proportional to the partial pressure of the gas above liquid
- Molar concentration of gas in liquid:

 $c = \alpha_B p$, α_B = Bunsen's absorption coefficient

- Solubility decreases with increasing T
- Water at 20 °C: for oxygen $\alpha_B = 0.031$, for nitrogen $\alpha_B = 0.016$. I.e. in water, higher amount of oxygen relative to nitrogen (compared to air)
- Nitrogen embolism: when air inhaled at high p → high amount of nitrogen dissolved → released when returned to atmosperic p → nitrogen bubbles in capillaries, fatal if in brain. Need slow return to surface or decompression chamber.

Solid in liquid

- Compounds that dissolve easily in polar solvent (water is strongly polar, dielectric $\epsilon_r = 80$):
 - dissociable into ions (e.g. NaCl)
 - organic compounds with polar groups
- Solubility of solids increases with increasing T
- If solution is saturated, any additional solid will form precipitate
- Can precipitate by lowering temperature
- Oversaturated solution can be obtained when solidification centers (e.g. impurities) are absent from solution



Properties of colloids

Colloidal dispersions

- Dispersive portion consists of particles of size 1 nm to 1 μ m
- Solid in liquid: sol; liquid in liquid: colloidal emulsion
- Examples: ink; milk
- Lyophilic colloids: disperse spontaneously, colloid has lower energy (solvation decreases surface tension). E.g. starch, protein in water.
- Lyophobic colloids are unstable (hydrophobic molecules aggregate). E.g. emulsion of oil in water; metal compounds in water.
- Tyndall effect: intense light scattering from a colloid, with intensity proportional to concentration. Most intense scattering for shorter wavelengths → blue color.

Electric double layer of colloidal particles

- Plays crucial role in stabilization of colloids
- Colloidal particles (macromolecules, micelles) have attractive Van der Waals interactions. If no repulsion, particles may coagulate.
- Repulsive Coulomb interaction due to electric double layer:
 - negatively charged ions from electrolyte dispersive medium are adsorped on the surface of particles
 - these attract positively charged counterions
 - overall, the double layer is neutral
 - slipping plane: part of the double layer that moves with the particle
 - zeta potential ζ : value of electrostatic potential at the slipping plane. Good stability of colloid for $\zeta \ge 40$ mV.



Electrophoresis

• In a uniform electric field *E*, the colloidal particles will move relative to the fluid dispersive medium, with velocity

$$v = \mu_e E$$

where μ_e is the electrophoretic mobility

- Smoluchowski equation: $\mu_e = \epsilon_r \epsilon_o \zeta / \eta$, where ϵ_r is the dielectric constant of fluid, η is the viscosity of fluid
- Zeta potential ζ also called electrokinetic potential
- Distinct colloidal particles have different μ_e , allows separation of particles (e.g. separation of albumin and globulin in blood serum)

Brownian motion

- Colloidal particles in a liquid or gas exhibit Brownian motion
- Due to imbalances of impact forces of molecules on the particle
- Understood before direct observation of molecules (Einstein, Smoluchowski)
- Kinetic energy of 1 collidal particle is of the order of $k_B T$





Sedimentation

- Due to gravity, particles of size r and density ρ_0 move through fluid with terminal velocity v
- For spherical particles: Stokes force $6\pi\eta rv$ balanced with buoyancy force $(4/3)\pi r^3(\rho \rho_0)g$, which gives the sedimentation velocity

$$v = \frac{2(\rho - \rho_0)gr^2}{9\eta}$$

- In equilibrium, a profile of colloid concentration is established, balance of sedimentation and diffusion
- Time needed to sediment: fast for coarse solutions, effectively infinite for analytical solutions
- Sedimentation rate test: aggregates of erythrocytes are formed during inflammation and sediment faster (bigger r).
- Centrifugation: enhanced sedimentation due to increased *g* (inertial force). Used e.g. to efficiently separate blood components.